

# Workshop in Cortona: Recent advances on cscK metrics

April 22-26, 2019

## 1 Summary

Finding canonical (Kähler-Einstein, cscK, extremal) metrics on compact Kähler manifolds is one of the central questions in complex geometry (see for example [2], [8], [7]). Given a Kähler metric  $\omega$  on a compact Kähler manifold  $X$ , one looks for a Kähler potential  $\varphi$  such that the curvature of the new metric  $\omega_\varphi := \omega + dd^c\varphi$  becomes very simple in some sense. This general problem is known to admit a solution in some important particular cases (theorems of Aubin-Yau, Yau, Chen-Donaldson-Sun to cite only a few) as well as obstructions (Matsushima, Futaki, Donaldson, etc.).

Recently there have been major breakthroughs related to a longstanding conjecture on the existence of cscK metrics and the properness of K-energy (see [6], [1], [3, 4, 5]) showing that a well-known necessary condition to the existence of cscK metrics is actually also a sufficient condition.

The aim of the workshop is to work especially on the last three papers in order to understand the techniques that could possibly lead to new developments in the subject.

## 2 Talks

1. Survey on cscK metrics and overview of the program (V. Guedj,  $\simeq$  1h).
2. Convexity of the K-energy along geodesics (M. Xia,  $\simeq$  3h).
3. A priori estimates in [3] (A. Deruelle & E. Di Nezza,  $\simeq$  6h).
4. The continuity method: openness (H. Auvray,  $\simeq$  2h).
5. Properness and existence of cscK metrics (H. Guenancia,  $\simeq$  3h).
6. Stability, properness and geodesic rays (S. Boucksom,  $\simeq$  3h).
7. Research talks (2h).

### 3 Schedule

## References

- [1] R. J. Berman, T. Darvas, and C. H. Lu, *Regularity of weak minimizers of the K-energy and applications to properness and K-stability*, arXiv:1602.03114, accepted in Annales scientifiques de l'ENS, (2018).
- [2] E. Calabi, *Extremal Kähler metrics*, in Seminar on Differential Geometry, vol. 102 of Ann. of Math. Stud., Princeton Univ. Press, Princeton, N.J., 1982, pp. 259–290.
- [3] X. Chen and J. Cheng, *On the constant scalar curvature Kähler metrics, apriori estimates*, arXiv:1712.06697, (Preprint 2017).
- [4] X. Chen and J. Cheng, *On the constant scalar curvature Kähler metrics, existence results*, arXiv:1801.00656, (Preprint 2018).
- [5] X. Chen and J. Cheng, *On the constant scalar curvature Kähler metrics, general automorphism group*, arXiv:1801.05907, (Preprint 2018).
- [6] T. Darvas and Y. A. Rubinstein, *Tian's properness conjectures and Finsler geometry of the space of Kähler metrics*, J. Amer. Math. Soc., 30 (2017), pp. 347–387.
- [7] G. Székelyhidi, *An introduction to extremal Kähler metrics*, vol. 152 of Graduate Studies in Mathematics, American Mathematical Society, Providence, RI, 2014.
- [8] S. T. Yau, *On the Ricci curvature of a compact Kähler manifold and the complex Monge-Ampère equation. I*, Comm. Pure Appl. Math., 31 (1978), pp. 339–411.